Math 315-003 Test 2 Name____________________

27–28 February 1 March 2004 Show relevant work!

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1. State and prove the product rule for derivatives.

2. For a natural number n, let $g(\mathrm{x}) = x^{1/n}$ for x > 0. Prove that $g'(x) = \frac{1}{n} x^{1/n-1}$.

3. State and prove the (Lagrange) Mean Value Theorem. State your assumptions.

4. Define uniform continuity and prove that a continuous function on a closed interval is uniformly continuous.

5. Let $F : (0,\infty) \rightarrow \mathbf{R}$ be such that $F'(x) = 1/x$ and $F(1) = 0$. Show $F(ab) = F(a) + F(b)$ for all $a, b > 0$.

6. Prove that a continuous function on a closed interval is integrable.

7. If f is continuous and bounded on an open interval (a,b), must f be uniformly continuous? Prove or give a counter-example.

8. Suppose f is differentiable on an open interval (a,b) with f' bounded. Prove f is uniformly continuous on (a,b).

9. For any numbers a and b and even natural number n, show that the following equation has at most two solutions.

$$x^n + ax + b = 0 \text{ for } x \text{ a real number}$$

What can you conclude if n is odd?

10. Let f, g be real valued functions defined for all real numbers so that each has n derivatives. Show:

$$(fg)^{(n)}(x) = \sum_{k=0}^{n} \binom{n}{k} f^{k}(x) g^{n-k}(x)$$